Effective theories for nonequilibrium matter: generalized time-reversal symmetry

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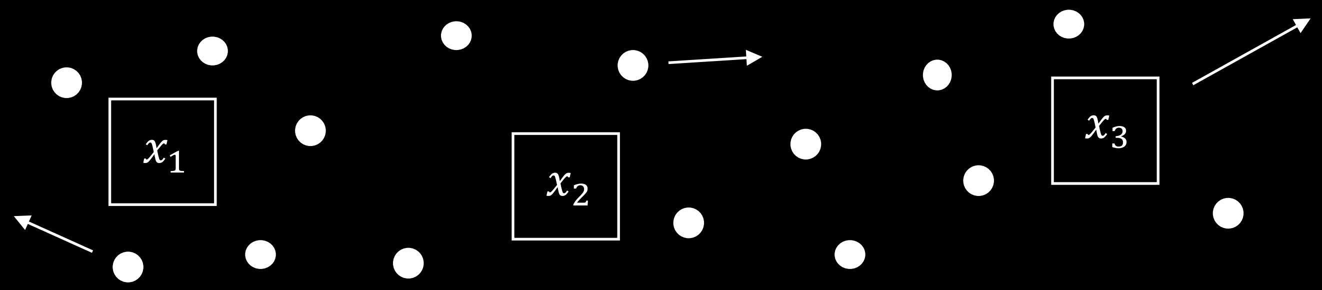
Time-reversal and dissipation

- Start with microscopic system of degrees of freedom $\mathbf{x} = \{x_a\}$ interacting with a bath (e.g. of molecules)



- Integrate out the bath (coarse-grain) and get stochastic equation:

$$\frac{dx_a}{dt} = f_a(\mathbf{x}) + \xi_a(t) \longleftarrow \begin{array}{c} \langle \xi_a(t) \rangle = 0 \\ \langle \xi_a(t) \xi_b(t') \rangle = 2Q_{ab}\, \delta(t - t') \end{array}$$

- What constraints should f and the noise variance Q obey if we want to demand time-reversal symmetry or break it in a controlled way?

Time-reversal and dissipation

- Equivalent: ***Fokker-Planck Equation*** for the PDF $P(x,t)$

$$\frac{dP}{dt} = -\frac{\partial}{\partial x_a}\left(f_a P - \frac{\partial}{\partial x_b}(Q_{ab}P)\right) \equiv -WP$$

- Has a ***stationary solution*** $P_{SS} = P(t \to \infty)$: $\quad WP_{SS}(x) = 0$

$$P_{SS}(x) \equiv e^{-\Phi(x)}$$

- *Result: Theory is compatible with microscopic TRS if*

$$W = e^{-\Phi} W^T e^{\Phi}$$

$$f_a = -Q_{ab}\,\partial_b \Phi(\mathbf{x}) + \partial_b Q_{ab} \qquad\qquad Q_{ab} = Q_{ba}$$

Effective theory

Our plan to describe a given system:

1. Write down a general Φ in a controlled "Wilsonian" expansion (e.g. gradient expansion, for field theories)
2. Build the most general W satisfying desired symmetries and
   $W = e^{-\Phi} W^T e^{\Phi}$

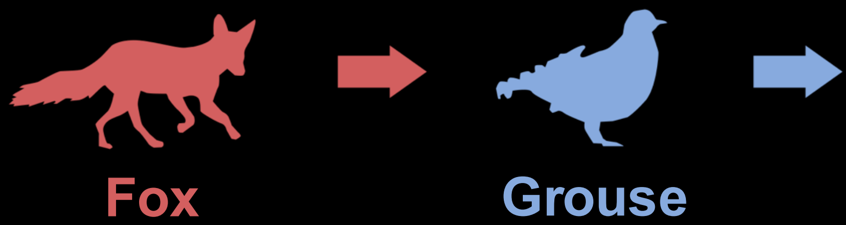
- e.g. add compatible dissipation to *any* Hamiltonian system

$$\partial_t \mathbf{S} = -\gamma \mathbf{S} \times \mathbf{B} - \underbrace{\lambda\, \mathbf{S} \times (\mathbf{S} \times \mathbf{B})}_{\text{Landau-Lifshitz-Gilbert Damping Term!}} + \boldsymbol{\xi}(\mathbf{S}, t)$$

- e.g. can read about *Noether's theorem* in our picture: how to enforce strong or weak conserved quantities as symmetries of W

2) Breaking time-reversal: nonreciprocity!

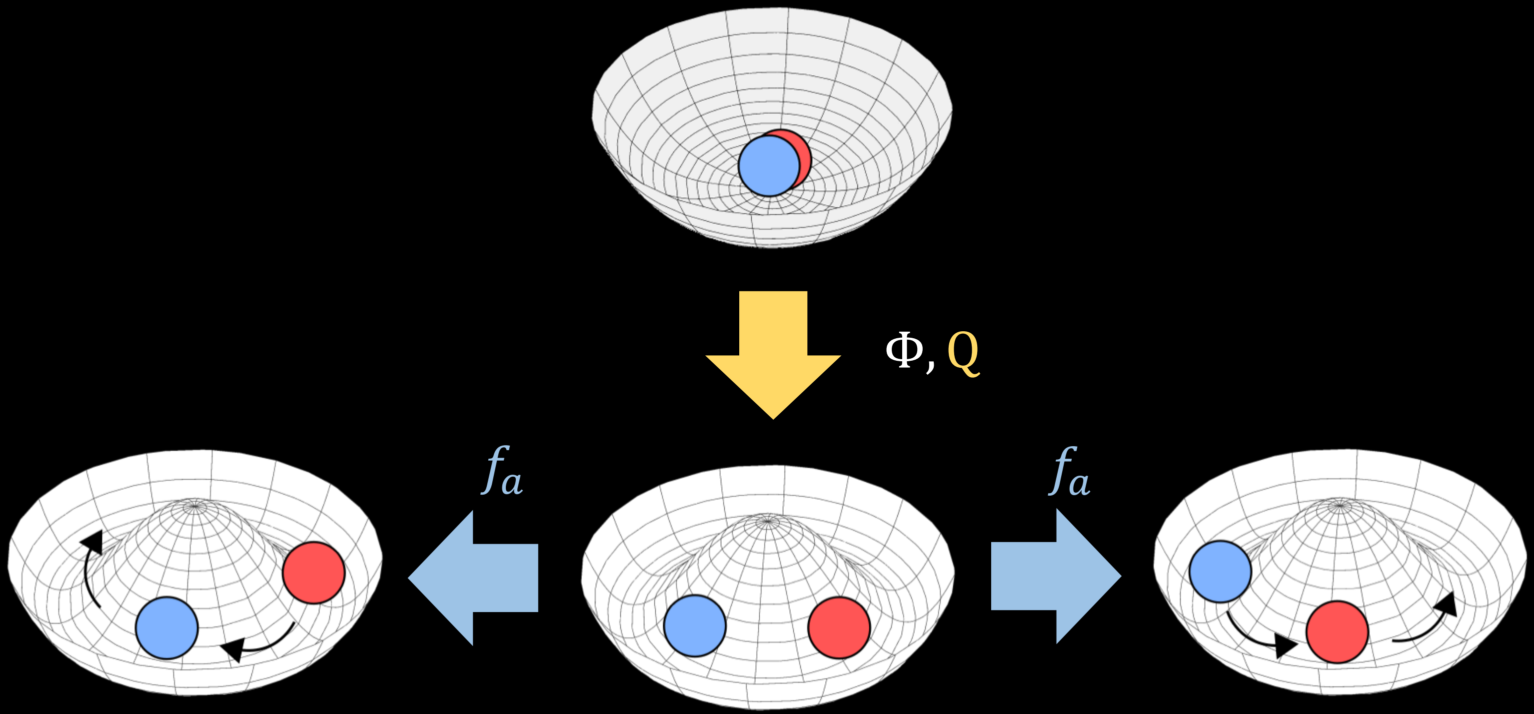
- In **odd** matter, interactions don't always follow Newton's third law! *e.g.* predator / prey dynamics



- **No detailed balance!** Need to also switch role of Grouse and Foxes! $gT = (\text{Fox} \leftrightarrow \text{Grouse}) + T$
- What if we want to include terms that are *odd* under time-reversal?
  - $W = e^{-\Phi} W^T e^{\Phi}$ $f_a = \partial_b V_{ab} - V_{ab} \partial_b \Phi$ $V_{ab} = -V_{ba}$
- Time-reversal breaking terms that still maintain the same stationary distribution P_{SS} !
  - e.g. odd elasticity (next talk: Isabella Zane)

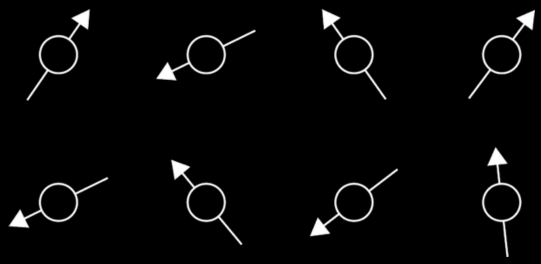
Dynamical phases

- These T-odd terms are how we capture T-breaking phases such as nonreciprocal Kuramoto model



A model of "flocking" spins

- Despite Φ unchanged, adding f_a terms can offer new ***dynamical*** universality classes!
- *e.g.* [Lew-Smith, **Farrell**, Qi, Friedman, Lucas *(to appear)*] studies a spin lattice model



$$\Phi = \int \mathrm{d}^2 x \frac{|\nabla\theta|^2}{Q}$$

NO true long range order even at low Q

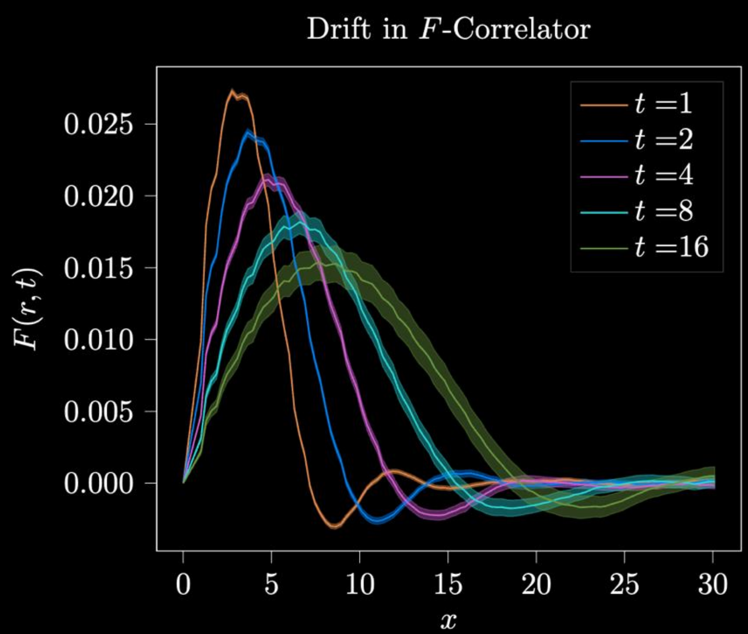
- Equation of motion:

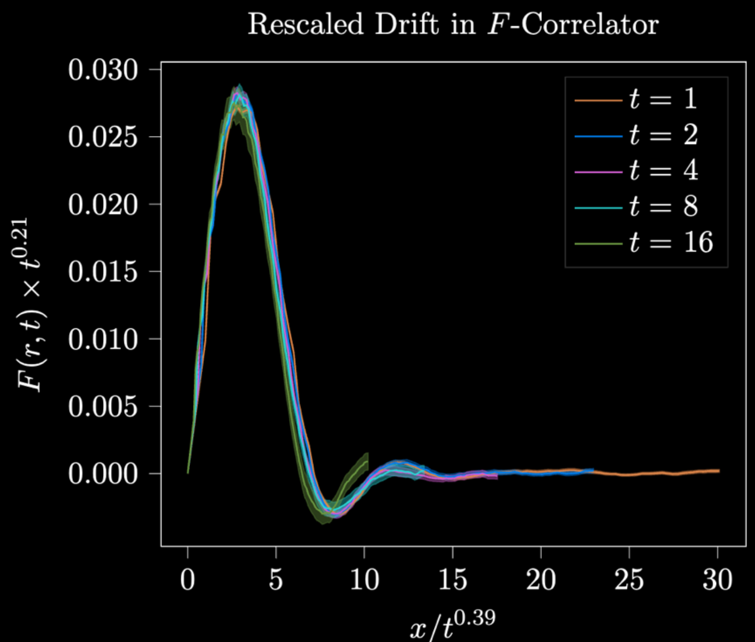
$$\partial_t \theta = D\,\nabla^2\theta \; + v_0[\cos\theta\,\partial_x\theta + \sin\theta\,\partial_y\theta] + O(\nabla^3)$$

At long wavelengths, this term is identical to constant-density flocking

A model of "flocking" spins—numerics

- Numerics: Dynamical correlator $F(r,t)$ which measures the correlation between a spin and one a distance r "ahead" of it at a time t later





$$F(r,t) = \frac{1}{t^{\eta(T)}} f\left(\frac{r}{t^{0.39}}\right)$$

Appendix: MSR Lagrangians

- For *field* theories, Lagrangian picture is more convenient
- Start with FPE: then *path integral*. Integrate in a *conjugate momentum* π_a:

$$Z = \int D\boldsymbol{\pi}\, D\mathbf{q}\, e^{i \int L\, dt}$$

$$L = \pi_a \partial_t q_a - \underbrace{W(\boldsymbol{\pi}, \mathbf{q})}_{\partial_a \to i\pi_a} = \pi_a \partial_t q_a - \pi_a f_a + i\pi_a \pi_b Q_{ab}$$

- TRS symmetry easy to implement!

$$\pi \to -\pi_a + \mathrm{i}\mu_a$$

$$\boxed{L = \pi_a \partial_t q_a - \pi_a v_a - \pi_a Q_{ab}(\mathrm{i}\pi_b + \mu_b)}$$

☺ Acknowledgments

1. Huang, **Farrell**, Friedman, Zane, Glorioso, Lucas*, Time-reversal symmetry and effective theories for nonequilibrium matter,* [2310.12233]

Xiaoyang
Huang

Aaron
Friedman

Isabella
Zane

Andy
Lucas

2. Lew-Smith, **Farrell**, Qi, Friedman, Lucas *(to appear)*

Elijah
Lew-Smith
(Brown)

Marvin
Qi