Endpoint of a Ballistic Dyakonov-Shur Instability

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Acknowledgments

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Rogue
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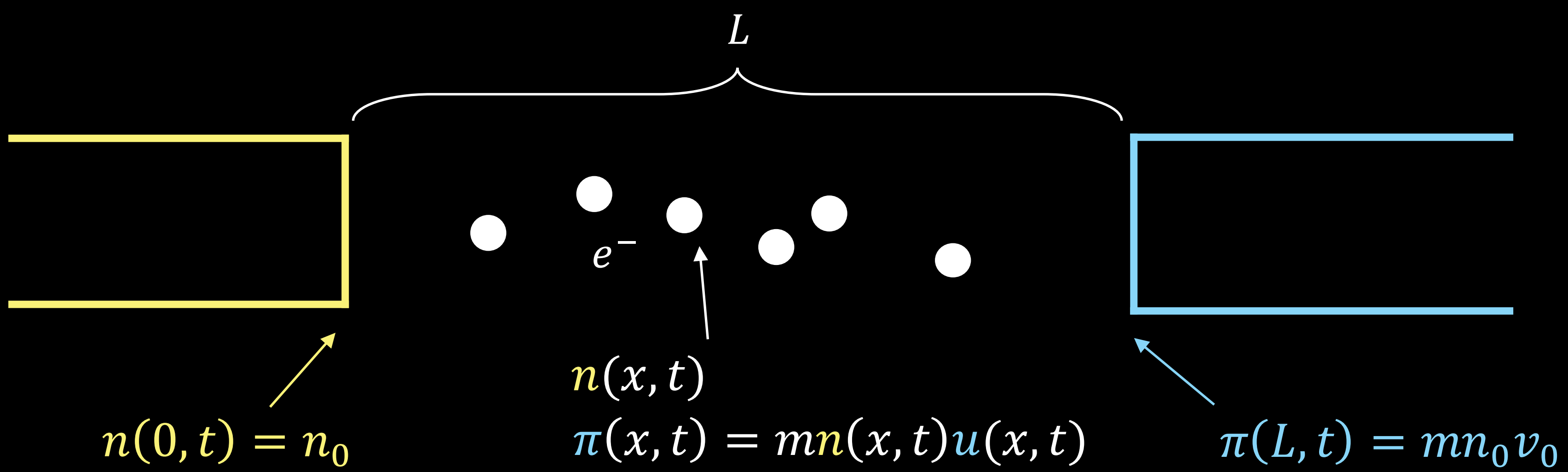
Dyakonov-Shur Instability



- [Dyakonov, Shur, 1993] :
  - normal modes $\delta n, \delta v \sim \mathrm{e}^{\mathrm{i}kx - \mathrm{i}\omega t}$
  - $\mathrm{Re}(\omega) = \frac{\pi s}{2L}$ —Terahertz!
  - $\mathrm{Im}(\omega) = \frac{s^2 - v_0^2}{2Ls} \log\left|\frac{s+v_0}{s-v_0}\right| > 0 \text{ if } v_0 < s$

Obstacles

- Boundary conditions (in particular $\pi(L,t) = \mathrm{const.}$)
- TeraHertz antennae… without this, $\mathrm{Power} \sim \mathrm{pW}$
- Momentum relaxation, $\mathrm{Im}(\omega) < 0 \text{ when } \gamma_{\mathrm{mr}} > v_0/L$ [Mendl, Polini, Lucas, 2022]
- ~~Crossover to ballistic physics~~

Today's punchline: ballistic transport should favour the DS instability; in particular, $\mathrm{Power} \sim \tau_{\mathrm{ee}}^2$ near instability onset

A nonlinear cartoon

- Take gas of fermions with $\varepsilon(\mathbf{p}) = \mathbf{p}^2/2m$
- They have a distribution function $f(\mathbf{x}, \mathbf{p}, t)$, which in equilibrium and near $T = 0$ is $f_0 = \Theta(\epsilon_F - p^2/2m)$

$$\partial_t f + \frac{\mathbf{p}}{m} \cdot \nabla f = C[f]$$

Hydrodynamic limit:
Local thermodynamic equilibrium consistent with conserved *particle number* and *momentum*

$$f(\mathbf{x}, \mathbf{p}, t) = \Theta\left(\mu(\mathbf{x}) - \mathbf{p} \cdot \mathbf{u}(\mathbf{x}) - \frac{\mathbf{p}^2}{2m}\right)$$

Ballistic limit:

$$C[f] = 0$$

$$\boxed{C[f] \approx \frac{1}{\tau_{ee}(\mathbf{p})}\left\{\Theta\left(\mu(x) - \mathbf{p} \cdot \mathbf{u}(x) - \frac{\mathbf{p}^2}{2m}\right) - f\right\}}$$

A nonlinear cartoon

- Parameterize:

$$f - f_0 \equiv \delta(\epsilon_F - \varepsilon) \times \sum_n a_n(x,t)\mathrm{e}^{in\theta} \qquad \begin{cases} n(x,t) = n_0 + \dfrac{m}{2\pi} a_0(x,t) \\ \pi(x,t) = \dfrac{m v_F}{2\pi} a_1(x,t) \end{cases}$$

- Giving

$$\partial_t a_n + \frac{v_F}{2} \partial_x (a_{n+1} + a_{n-1}) = C_n$$

- Final ingredient: presence of background flow u_0

$$\partial_t \to \partial_t + u_0 \partial_x$$

A nonlinear cartoon

$$\partial_t a_0 + u_0 \partial_x a_0 + v_F \partial_x a_1 = 0$$

$$\partial_t a_1 + u_0 \partial_x a_1 + \frac{v_F}{2} \partial_x (a_2 + a_0) = 0$$

$$\partial_t a_2 + u_0 \partial_x a_2 + \frac{v_F}{2} \partial_x (a_3 + a_1) = 0 - \frac{1}{\tau_{\mathrm{ee}}} a_2 - \frac{1}{\tau_{\mathrm{ee}}} \left(\frac{a_0^2}{m v_F^2} + \frac{4 a_1^2}{m v_F^2} \right)$$

$$\vdots$$

$$\partial_t a_n + u_0 \partial_x a_n + \frac{v_F}{2} \partial_x (a_{n+1} + a_{n-1}) = 0 - \frac{1}{\tau_{\mathrm{ee}}} a_n$$

at lowest order in τ_{ee}, right hand side vanishes, and one can show:

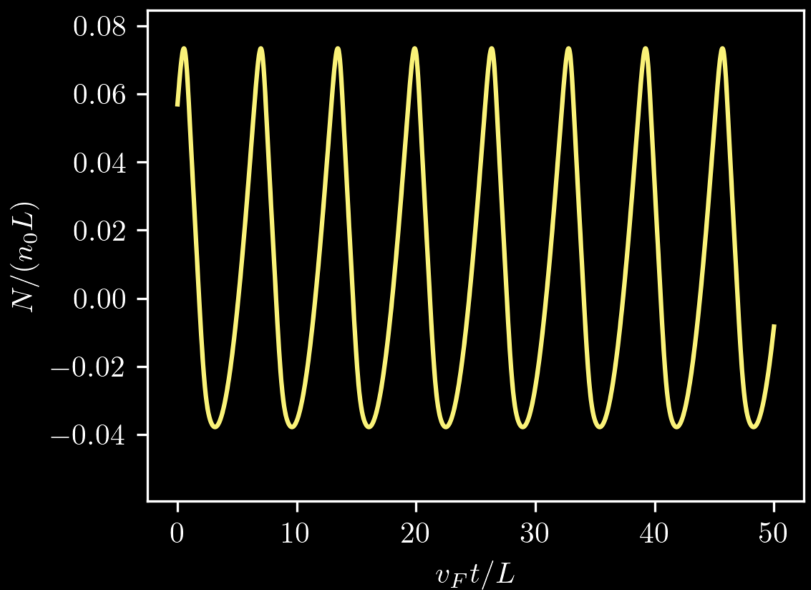
$$\left.\begin{aligned} \partial_t n + \frac{1}{m} \partial_x \pi &= 0 \\ \partial_t \pi + \partial_x \left(\frac{\pi^2}{m n_0} \right) + \partial_x \left((2\pi\hbar^2) \frac{n^2}{4\pi} \right) &= 0 \end{aligned}\right\}$$

Hydrodynamics in the "Boussinesq" approximation

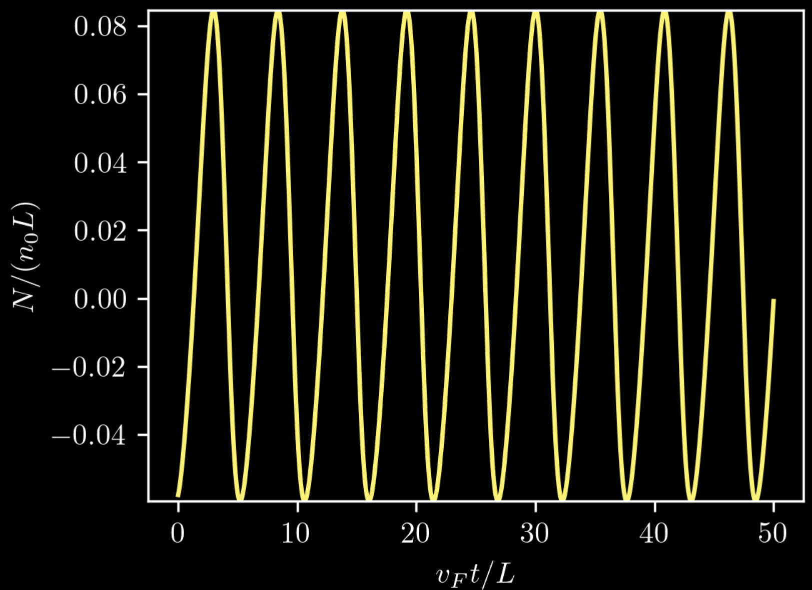
Numerics

- Truncate at $n_{\max} = 40$

$$\frac{\tau_{\mathrm{ee}} v_F}{\mathrm{L}} = 0.25$$

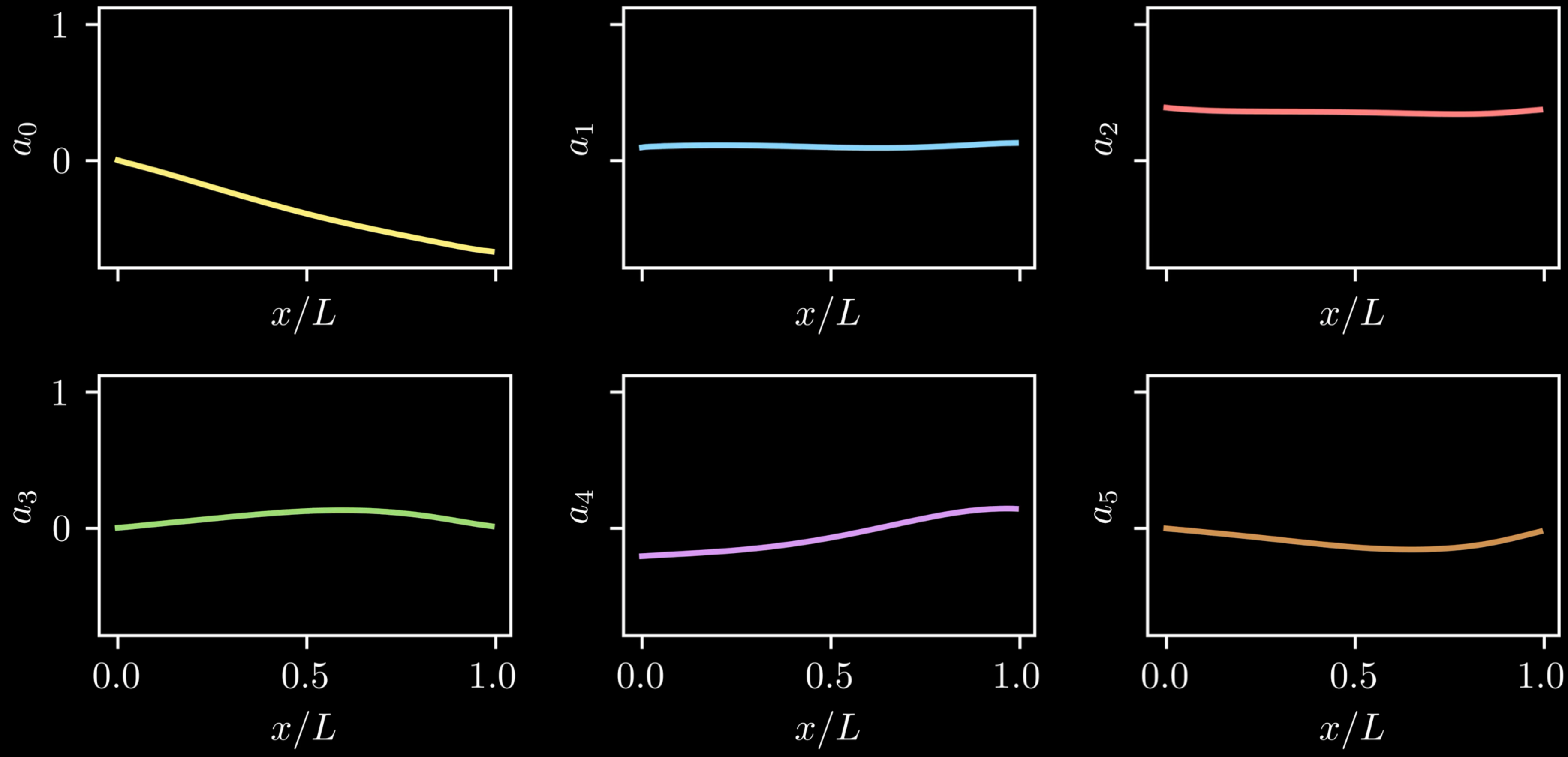


$$\frac{\tau_{\mathrm{ee}} v_F}{\mathrm{L}} = 1.25$$



Numerics

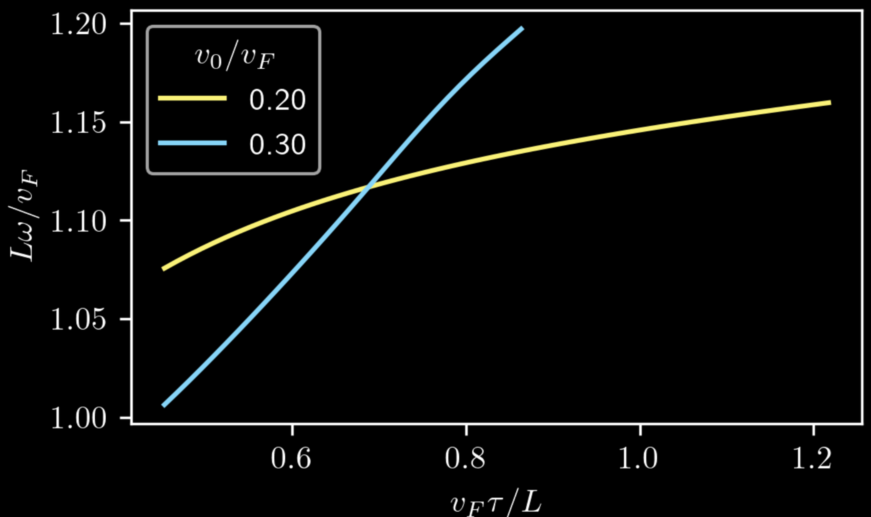
$$\frac{\tau_{\mathrm{ee}} v_F}{\mathrm{L}} = 1.25$$

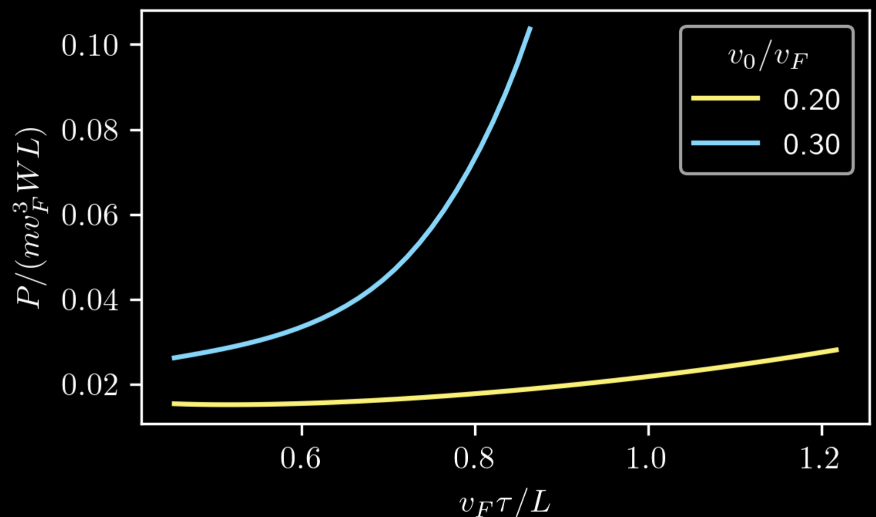


Characterization

- Imagine the device is separated from a metallic gate by an oxide, forming a capacitor
- Power from charge-discharge cycles:

$$\text{Power} \approx \frac{1}{T}\left|\int_0^T \frac{\mathrm{d}t}{T}\frac{(eN)^2}{2C}\cos(2\pi t/T)\right|$$





Conclusions & Outlook

- ballistic transport should favour the DS instability
- Illustrated in the context of a quadratic toy model for the ballistic-hydrodynamic crossover

More ideas:

- Nonlinear treatment with temperature gradients in and around hydro regime
- Nonlinear transport through 2D geometries

- Thanks for listening! ☺
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