Hydrodynamics with Helical Symmetry

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09 March, 2023

(arXiv: 2208.12269)

Collaborators ☺

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Outline

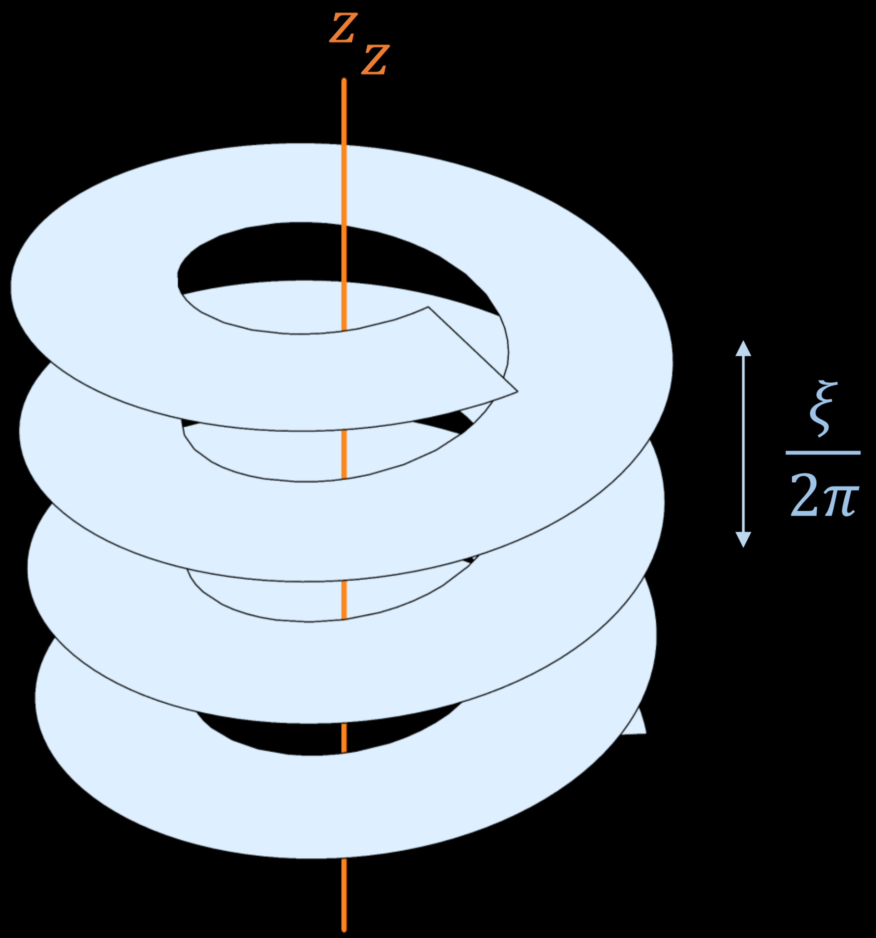
- Helical symmetry
- Review Hydrodynamics
- Helical Hydrodynamics
- Outlook

Helical Symmetry

- Only a *combination* of translation and rotation is a symmetry
- Corresponding conserved quantity:

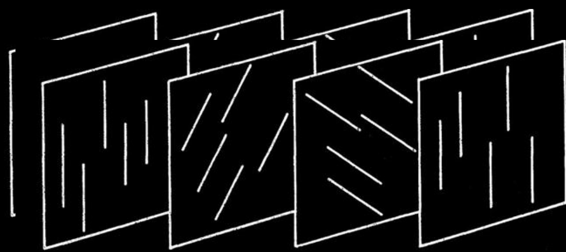
$$K_z = P_z + \frac{L_z}{\xi}$$

- Mixes spacetime symmetries



Realizations:

- Cholesteric liquid crystals (pinned)



[Lubensky, *Hydrodynamics of Cholesteric Liquid Crystals*, 1972]

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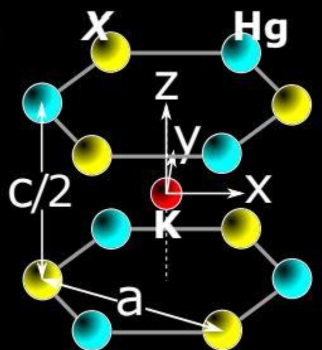
- Non-symmorphic crystal (“discrete helical symmetry”)



KHg*X*

[Wang et al, *Hourglass Fermions*, 2016]

Review of hydrodynamics

- Follow a small number of quantities that are conserved, (P_x, P_y, N)

$$\frac{d}{dt}\int n \, \mathrm{d}\mathbf{x} = 0$$

$$\partial_t n + \partial_i J_i = 0,$$

$$\partial_t \pi_A + \partial_i \tau_{iA} = 0.$$

- But what are $\boldsymbol{\tau}, \mathbf{J}$, and how are they related to $n, \boldsymbol{\pi}$?
- This talk: write down all the terms you can consistent with the symmetries. (Can check within Kinetic Theory and modern EFT framework)

Gradient expansion

- Expand $\boldsymbol{\tau}$ in powers of the hydrodynamic variables and in powers of ∇ (using $\mathbf{v}$ instead of $\boldsymbol{\pi}$).
- (linear response is sufficient)

$$\tau_{iA} = A_{iA} n + B_{iAB} v_B + C_{iAk} \partial_k n + D_{iAkB} \partial_k v_B + \cdots$$

$$\tau_{iA} = A_{iA} n + B_{iAB} v_B + C_{iAk} \partial_k n + D_{iAkB} \partial_k v_B + \cdots$$

- New *dissipationless* terms allowed!

$$C_{iAk} = v_s^2 (\alpha \epsilon_{iAz} \delta_{kz} + \beta \epsilon_{iAz})$$

$$\tau_{AB} \sim v_s^2 \alpha \epsilon_{AB} \partial_z n$$

$$\tau_{zA} \sim v_s^2 \beta \epsilon_{AB} \partial_B n$$

Same game for $\mathbf{J}$:

$$J_A = n_0 v_A + b\epsilon_{AB}\partial_z v_B$$

$$J_z = n_0 v_z + a\epsilon_{AB}\partial_A v_B$$

- It turns out, for general thermodynamic reasons:

$$a = -\alpha$$

$$b = -\beta$$

Key Terms in Equations of motion:

$$n_0\partial_t v_A + v_s^2\,\partial_z n + v_s^2(\alpha+\beta)\epsilon_{AB}\partial_B\partial_z n - \eta\partial_i\partial_i v_A - \eta_z\partial_z^2 v_A = 0$$

$$n_0\partial_t v_A + v_s^2\,\partial_z n + v_s^2(\alpha+\beta)\epsilon_{AB}\partial_B\partial_z n - \eta\partial_i\partial_i v_A - \eta_z\partial_z^2 v_A = 0$$

$$\partial_t n + n_0\partial_i v_i - (\alpha+\beta)\epsilon_{AB}\partial_A\partial_z v_B = 0$$

What about π_z?

Why no *rotational viscosity* η_o?

What is the value of α, β ?

What about π_z?

$$\partial_t \pi_z + n_0 \partial_z \mu = -\frac{1}{\xi}(\tau_{xy} - \tau_{yx})$$

- If there is any part of $\tau_{xy} - \tau_{yx}$ that is *not* a total derivative, π_z is not conserved!

Rotational viscosity relaxes π_z

- When angular momentum not conserved:

$$\tau_{xy} - \tau_{yx} = -2\eta_\circ \underbrace{\left(\partial_x v_y - \partial_y v_x\right)}_{\omega_z}$$

- But a fluid with helix symmetries in in equilibrium with density matrix:

$$\rho \sim e^{-\beta(H - \mu N - v_A P_A - w K_z)}$$

- A “boost” in K_z $(w \rightarrow w + u)$ leaves the fluid in equilibrium, so it can not create any entropy!

Rotational viscosity relaxes π_z

$$\omega_z \to \omega_z + 2u/\xi$$

- Not good enough! Instead we need:

$$\tau_{xy} - \tau_{yx} \sim -4\eta_\circ(\omega_z/2 - v_z/\xi)$$

- This gives us the required “not total derivative term”, leading to:

$$\partial_t \pi_z + n_0 \partial_z \mu = -\frac{4\eta_\circ}{\rho\xi^2}\,\pi_z + \frac{2\omega_z}{\rho\xi}$$

$$\gamma = \frac{4\eta_\circ}{\rho\xi^2}$$

Same relaxation rate as calculated in kinetic theory and EFT!

α is fixed!

- After v_z has relaxed:

$$\tau_{xy} - \tau_{yx} \sim -n_0 \xi \partial_z \mu / 2$$

$$\alpha = -n_0 \xi / 2$$

Summary and Outlook

- Mixing of spacetime symmetries has interesting consequences for hydrodynamics
- Landau, Kinetic Theory, EFT (not presented) all offer useful perspectives on the complications this mixing provides

- “discrete helix”: interesting?
- Other theories that mix multipolar symmetries and translations?

Thanks a lot for listening!