2D Electron Transport in Complex Geometries

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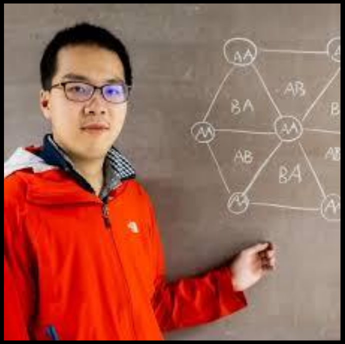
Acknowledgments

Andy Lucas
(Boulder)



Canxun Zhang

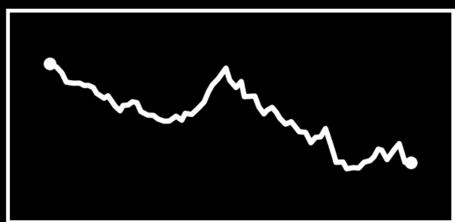
Ludwig Holleis

Rogue
(Cat)

Andrea Young
(UCSB)

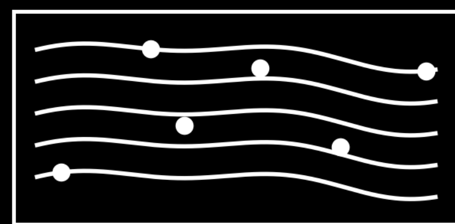
Overview

- Recall (from the previous talk) that current flow patterns (transport) are sensitive to the type and strength of different physical processes



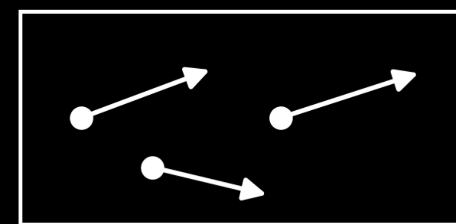
Diffusive

γ_{mr}



Hydrodynamic

$\gamma_{mr} \ll \gamma_{mc}$



Ballistic

$\gamma_{mr} = \gamma_{mc} = 0$

- But real materials often operate in *intermediate* regimes
- How does one use transport to determine parameters of this kind *quantitatively*?
- Imaging is awesome, but it takes a long time and feels like overkill to determine $O(1)$ phenomenological parameters…

Current fractions in a multiterminal geometry

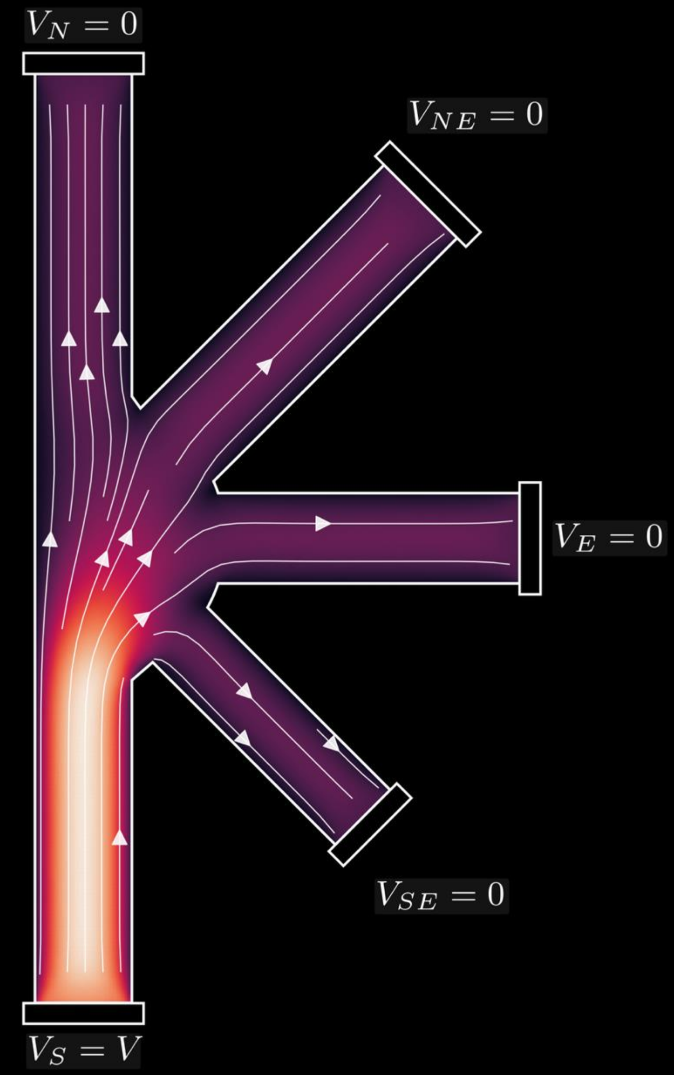
[**JF**, Lucas, 2605.03030]

- Ballistic v.s. hydrodynamic, diffusive?
  - Ballistic carriers travel in semiclassical trajectories, so I_N should be large
- Hydrodynamic vs diffusive?
  - Recall expectations for resistance for rectangle with ℓ, w

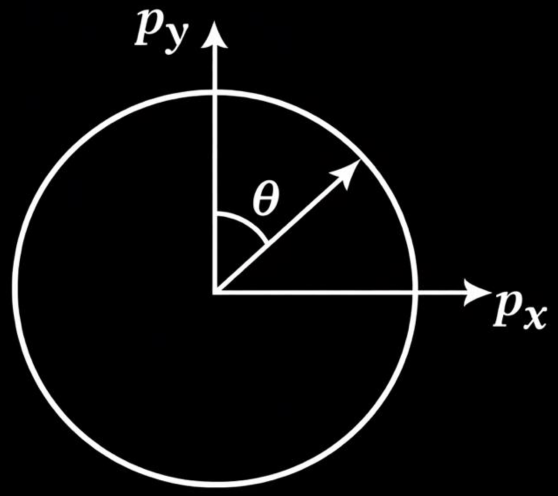
$$R_{diff} \sim \gamma_{mr} \frac{\ell}{w}$$

$$R_{hydro} \sim \gamma_{mc}^{-1} \frac{\ell}{w^3}$$

  - So create channels with fixed $\frac{\ell}{w}$ but varying w!



A "toy" kinetic theory



- Linear Botzmann equation
- Take fermions with $\varepsilon(|p|)$ (circular Fermi Surface)
- They have a distribution function $f(\mathbf{x}, \mathbf{p}, t)$, which in equilibrium and near $T = 0$ is $f_0 = \Theta(\epsilon_F - \varepsilon)$

$$\partial_t f + \mathbf{v}(\mathbf{p}) \cdot \nabla f = C[f]$$

- Then parameterize deviation from equilibrium:

$$\delta f = \phi(\mathbf{x}, \mathrm{t}, \theta)\ \delta(\epsilon_F - \varepsilon) \qquad C \to W[\phi(\theta)]$$

Harmonics capture conserved quantities:

$$\phi(\mathbf{x}, \mathrm{t}, \theta) = \frac{a_0}{2} + \sum_n [a_n \cos n\theta + b_n \sin n\theta]$$

- Boltzmann becomes a tower of coupled PDEs

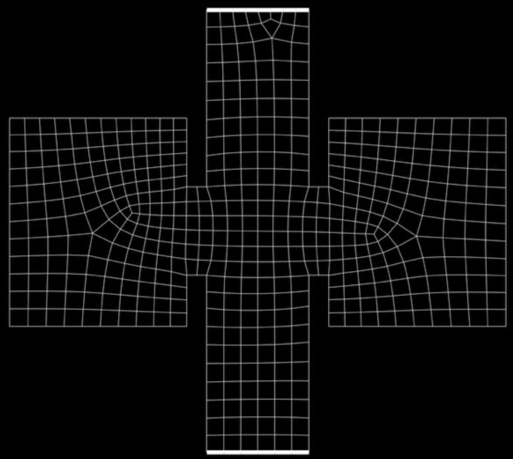
$$\partial_t a_0 + v_F(\partial_x a_1 + \partial_y b_1) = 0$$

$$\partial_t a_1 + \frac{v_F}{2}\big(\partial_x(a_2 + a_0) + \partial_y(b_2 - b_0)\big) = -\gamma_{\mathrm{mr}}\, a_1$$

$$\partial_t a_n + \frac{v_F}{2}\big(\partial_x(a_{n+1} + a_{n-1}) + \partial_y(b_{n-1} - b_{n+1})\big) = -(\gamma_{\mathrm{mr}} + \gamma_{\mathrm{mc}})\, a_n$$

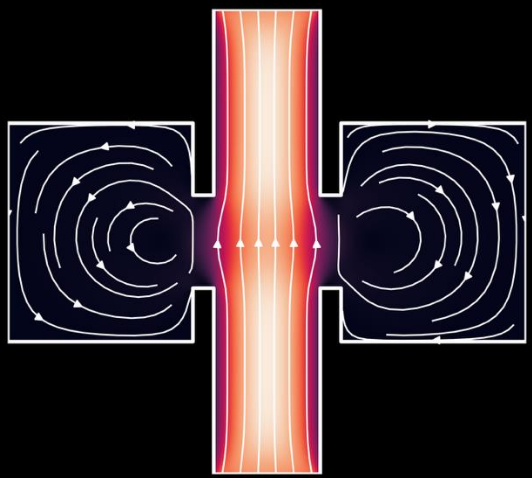
Solve numerically in any geometry you want!

Drain
$V = 0$



Source
$V = V_S$

High-order Discontinuous Galerkin Spectral Element Method



$\gamma_{mc} = 30.0$
$\gamma_{mr} = 0.5$

- Have studied wall boundary conditions diffuse, specular, or a blend
- Available at github.com/jackhfarrell/FermiSea.jl

[Zhang, Redekop, Stoyanov, **JF**, *et al.* 2603.11175]

Reminder: Current fractions in a multiterminal geometry

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- Hydrodynamic vs diffusive?
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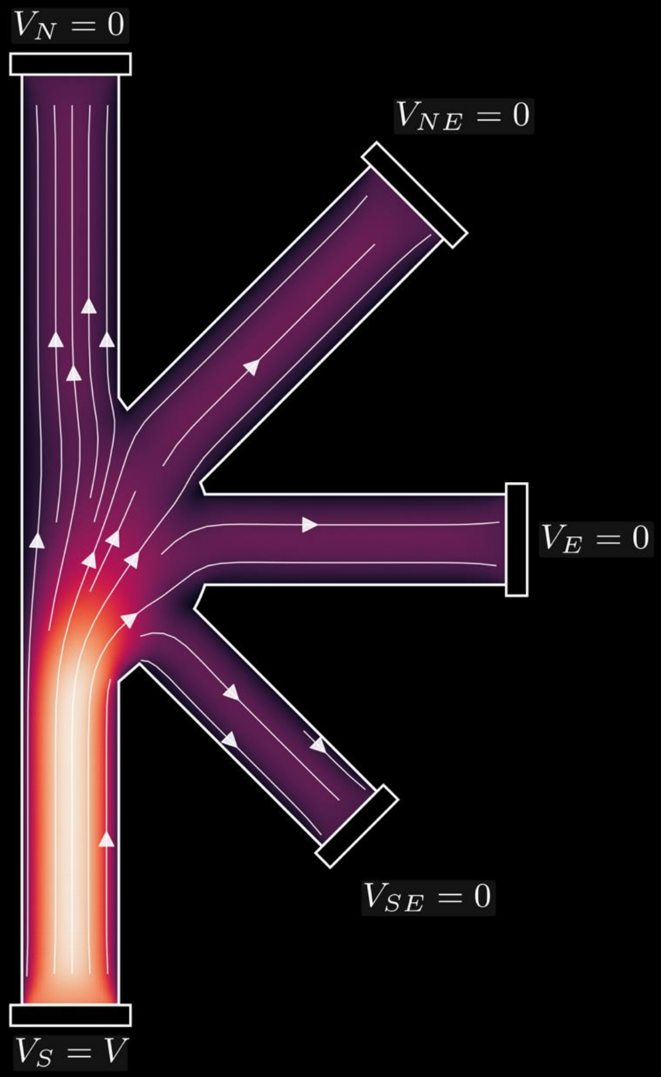
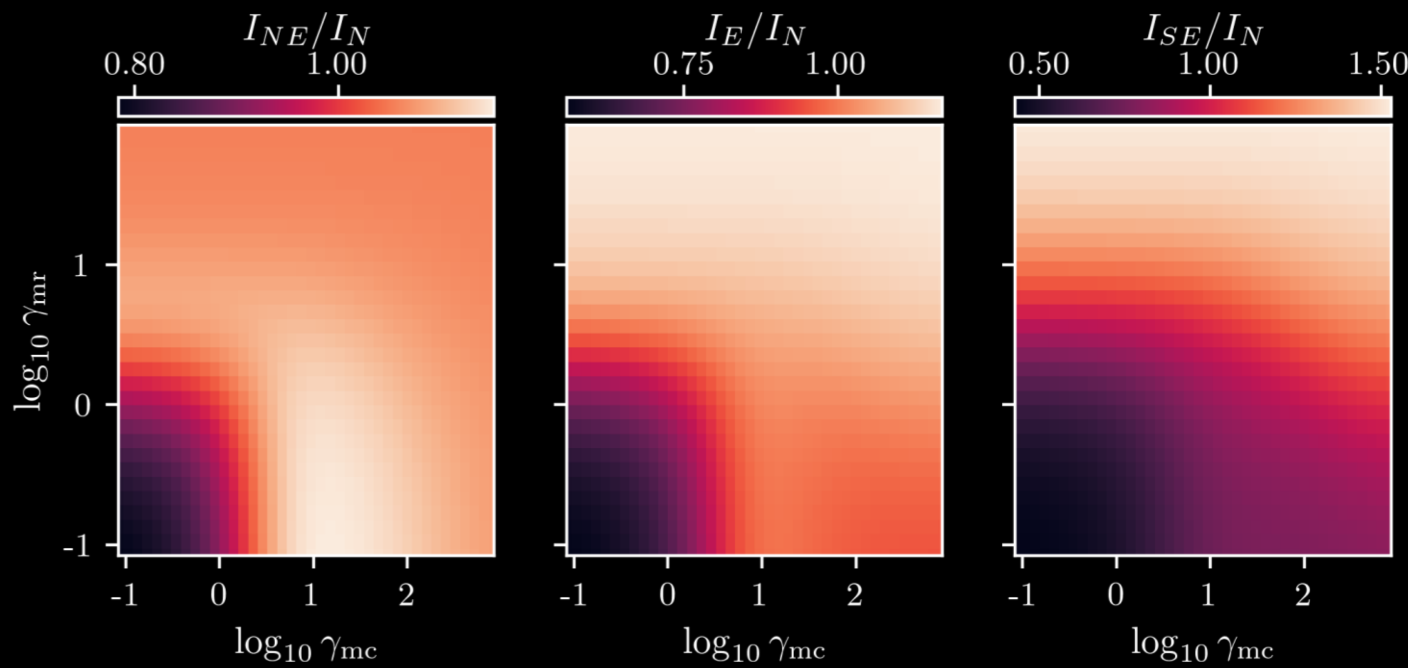
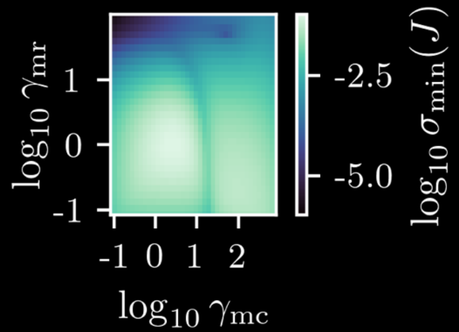


Ballistic-Hydrodynamic-Diffusive Crossovers



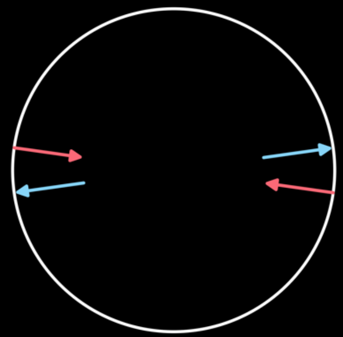
- If we were given a precise measurement of these three fractions, could we infer γ_{mr}, γ_{mc} ?
- Take take singular values of Jacobian of map $\gamma_{mr}, \gamma_{mc} \mapsto I_{NE}, I_E, I_{SE}$



Electron-electron collisions

- If our momentum-conserving collisions are short-range electron-electron scattering near $T = 0$, the collision integral $W[\phi]$ has very nice structure!

- Consider $\mathbf{p}_1, \mathbf{p}_2 \to \mathbf{p}_3, \mathbf{p}_4$, all four should live on FS, and conserve momentum and energy?



- Inversion symmetry: $W[\phi(\theta) - \phi(\pi - \theta)] \sim 0$

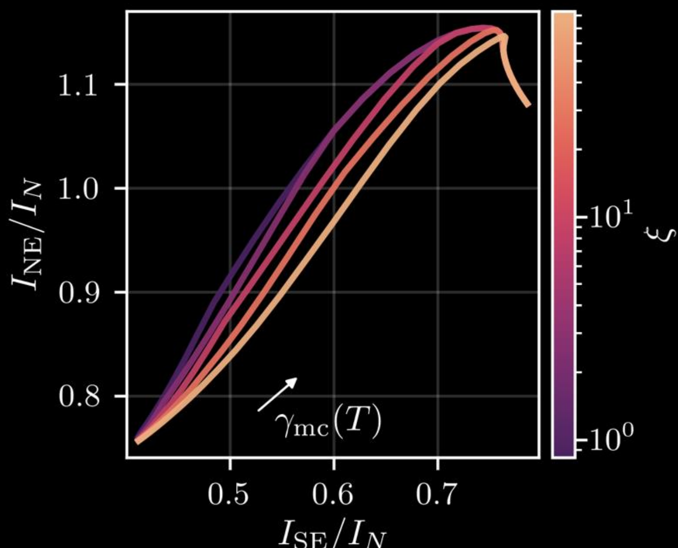
$$W[\phi(\theta) + \phi(\pi - \theta)] \sim \gamma_{mc}$$

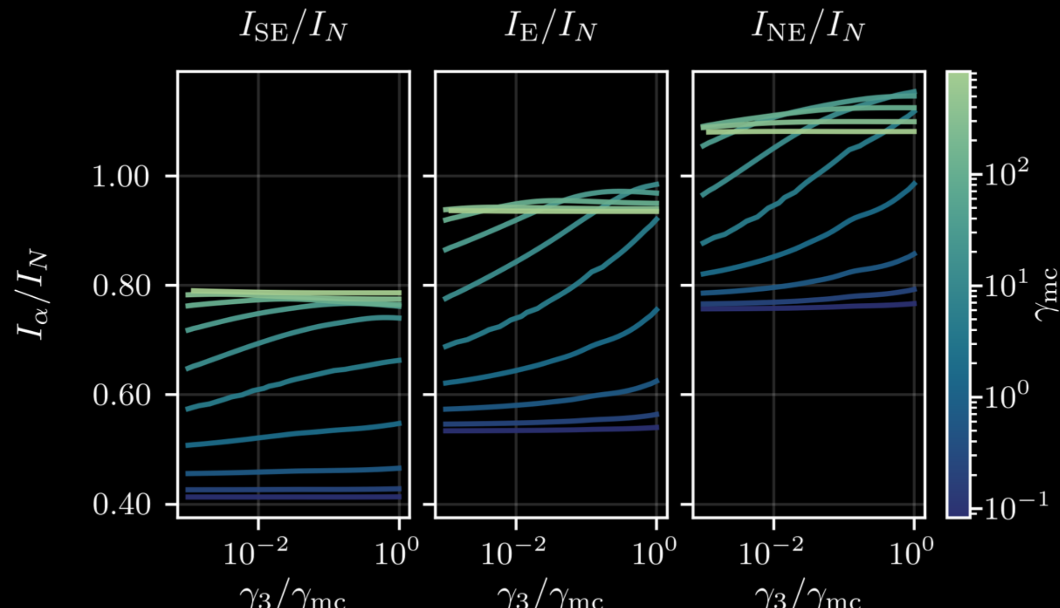
- Slightly better: remember this Fourier expansion?

$$\gamma_m = \begin{cases} \gamma_{mc} & m > 1 \text{ even} \\ \gamma_3 \left(\frac{m}{3}\right)^4 & m > 1 \text{ odd} \end{cases}$$

Sensitive to mode-resolved decay rates

- Expectation: $\gamma_{mc} \sim T^2$, $\gamma_3 \sim T^4$
- Parameterize $\gamma_3 \sim \xi\gamma_{mc}$
- Imagine sweeping T

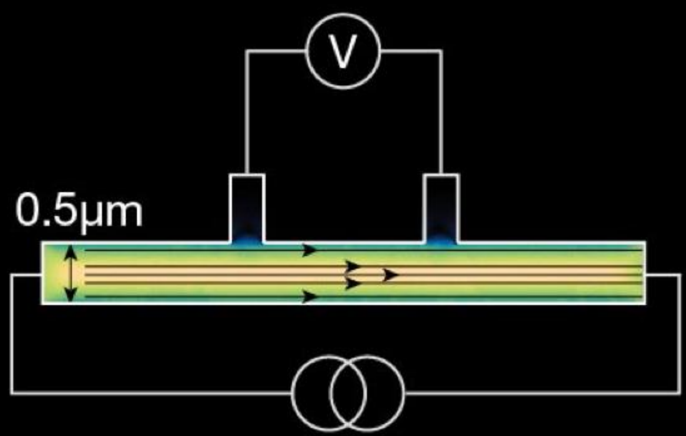


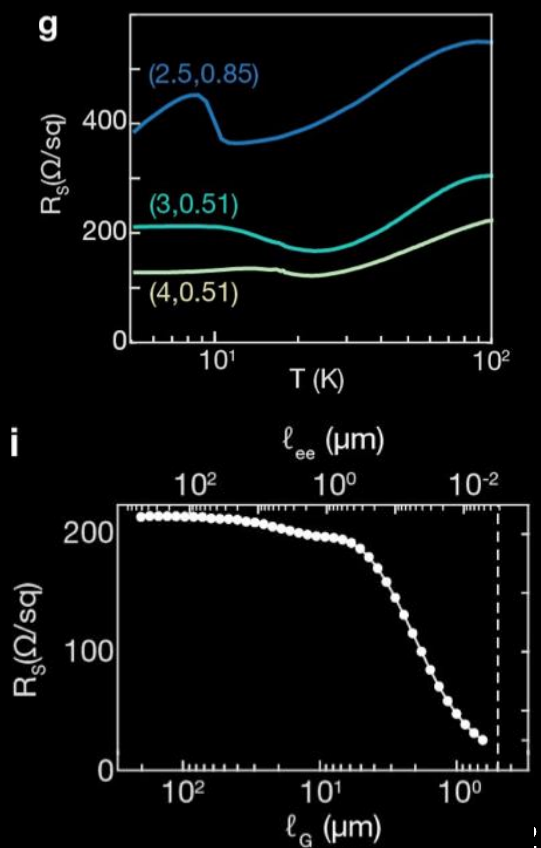


Proof of concept?

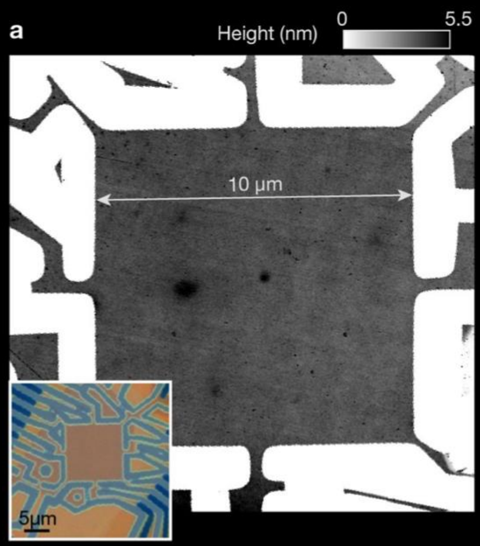
Holleis,Choi, Zhang, **JF**, *et al.* 2604.21912]

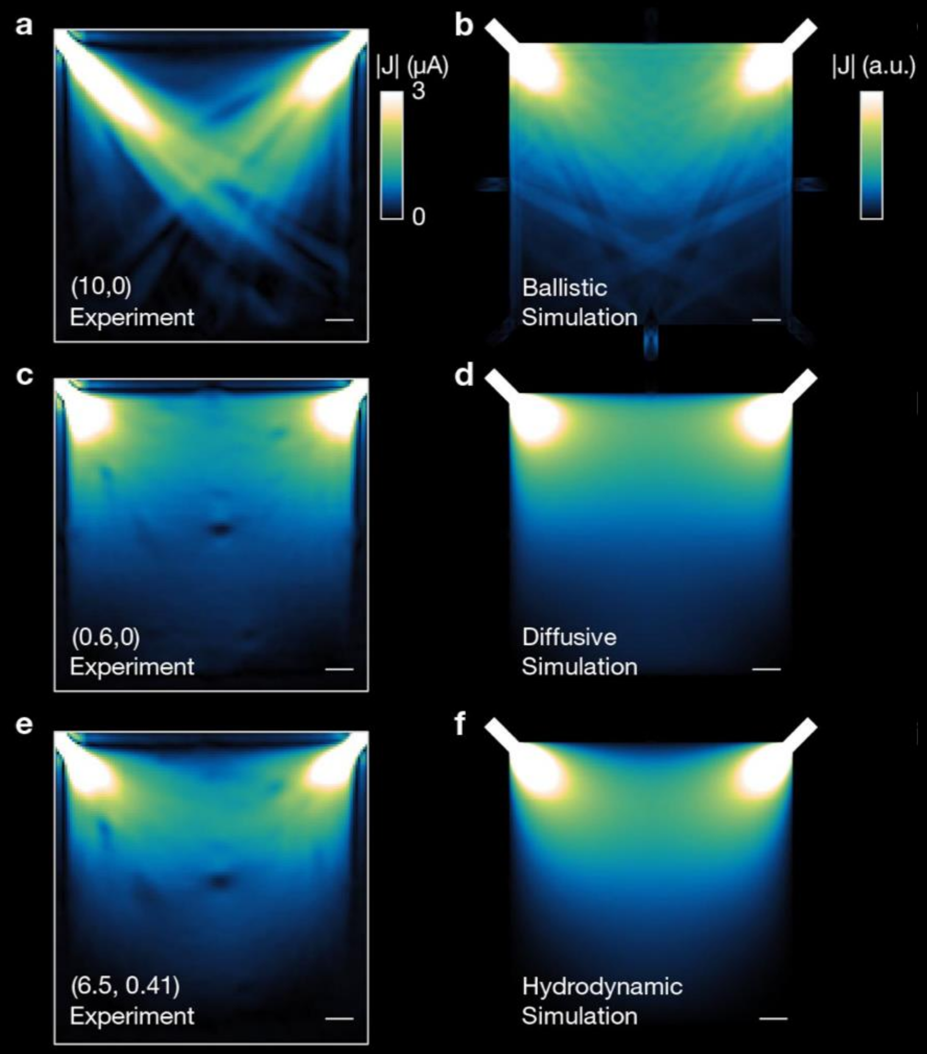
- How are we doing in terms of modeling these transport observables?
- Extremely large device, 13 layer rhombohedral stacked graphene!





Proof of concept in even bigger device?

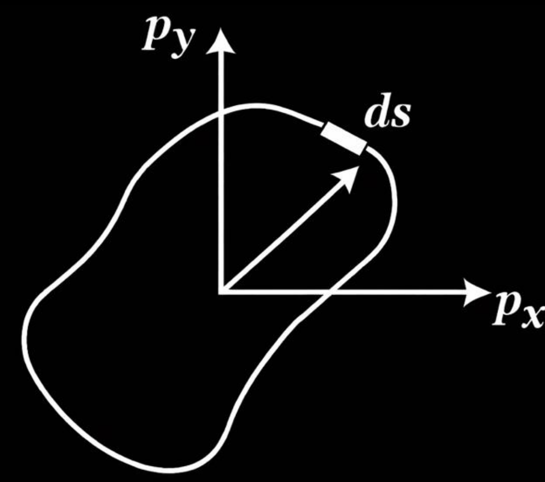




Future direction: anisotropic FS?

- Imagine single Fermi surface of some arbitrary shape parameterized by arclength s

$$\delta f = \phi(\mathbf{x}, \mathrm{t}, \mathbf{p})\, \delta(\epsilon_F(s) - \varepsilon)$$



$$\varepsilon(\mathbf{p}) = \epsilon_F$$

- Want to expand ϕ in functions

$$\phi(\mathbf{x}, \mathrm{t}, s) = \sum_n a_n(\mathbf{x}, t)\, \chi_n(s)$$

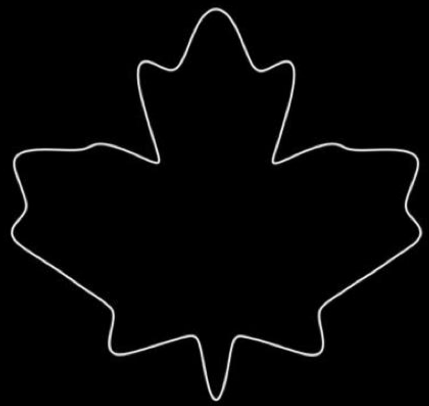
$$\int \frac{\mathrm{d}^2\mathbf{p}}{(2\pi)^2} \chi_n(s)\, \chi_m(s)\, \delta(\epsilon_F(s) - \varepsilon) = \int \frac{\mathrm{d}s}{(2\pi)^2} \chi_n(s)\, \chi_m(s) \frac{1}{v_F(s)}$$

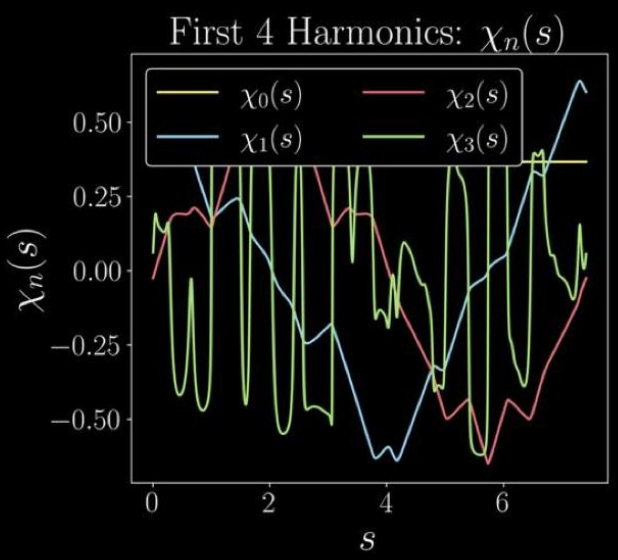
$$= \delta_{nm}$$

- Boltzmann becomes a vector advection equation:

$$\partial_t \mathbf{a} + \partial_x(L_x \mathbf{a}) + \partial_y(L_y \mathbf{a}) = \mathbf{W}[\mathbf{a}]$$

Example / outlook





- Should be very interesting to try and model space resolved transport in materials w/ more complicated FS (including honest collision integral, see the methods of next talk!)
- In particular, can you engineer geometries in 2D (or 3D) to try and measure whichever parameter (rotational viscosity?)
  - inverse design?

Thanks for your attention! ☺